Changes in the Rate of Profit, the Capital-Output Ratios and Relative Prices: An Alternative View

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Introduction

Throughout most of the literature on Sraffa's prices of production (Sraffa, 1960) the rate of profit has been treated as an exogenous variable, and by setting the value of one of the distributive variables, we can determine the system of relative prices and the the other distributive variable in terms of a chosen numéraire (Pasinetti, 1977). In an earlier empirical application of a Sraffa-Pasinetti model (del Valle, 1992), I have placed the particular behaviour of these prices at the centre of the explanation of the changes in the structure of the economy defined in terms of the capital-output and capital-labour ratios.

In the usual Sraffian analysis every time the system of relative prices has been calculated, the rate of profit has been allowed to affect all the sectors of the economy at the same time, thus being unable to distinguish the effects of the increase in the price of any particular commodity whose individual rate of profit increased, so that the final movement may be the combined effect of changes in its own price and changes in the prices of all the other commodities for which the rate of profit also increased.

To "separate", as it were, the individual, industry by industry, effects of the change in the rate of profit, we will formalize a model in which the rate of profit is allowed to affect prices one industry at a time. We will analyze the behaviour of prices when, in a first stage,

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the rate of profit is increased from 0 to maximum level R in only one industry. Once this is done, we will analyze what will happen in the second stage, when the rate of profit is changed from 0 to R in a second industry keeping r = R in the first industry and r = 0 elsewhere.

At the and of this paper we will present a brief empirical application, using an aggregate 10 x 10 input-output data for the Puerto Rican economy for 1967.

The Model

To begin let us rewrite the system of equations for the relative prices by postmultiplying the technical coefficients matrix **A** by a diagonal matrix changing r only in the first industry. We then let that value of r increase from 0 to the system's maximum R and observe the behaviour of ***all*** prices in terms of the value of the capital goods in that sector for which the rate of profit was changed (i.e. $\mathbf{pA}_1$). That is, we will write:

$$\boldsymbol{p} = \boldsymbol{a} \cdot w + \boldsymbol{pA} + \boldsymbol{pA} \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \qquad \textbf{(1)}$$

and after rearranging this equation for **p** we divide the whole expression by $\mathbf{pA}_1$. Bear in mind that although we are allowing the rate of profit to be positive only in sector one, *all* prices will be affected. What is important to realize is that such changes must be the result of the increase in the rate of profit in that particular sector.

When the rate of profit had reached its maximum in sector 1, we then allow the next sector to have a positive rate of profit, keeping the first sector's rate at the maximum and all the others equal to zero:

$$\boldsymbol{p} = \boldsymbol{a} \cdot w + \boldsymbol{pA} + \boldsymbol{pA} \begin{bmatrix} R & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \tag{2}$$

Again, we increased the rate of profit in sector 2 from 0 to its maximum R, which is the same as for the first sector. We observe the new behaviour of all prices, still measuring prices in terms of the ***first*** sector's value of capital, so as to be able to make a straightforward comparison (i.e. keeping the same numéraire), as well as to allow us to analyse relative price changes in terms of changes in relative capital-output ratios, amongst other things. We repeated this procedure until each sector's rate of profit had been turned positive and allowed to reach the maximum value, R.

We can formalize this analysis, for the first Stage, in the following way:

$$\begin{aligned} p_1 &= a_1 \cdot w + \boldsymbol{p}(1 + r_1)\boldsymbol{A_1} \\ \boldsymbol{p_i} &= \boldsymbol{a_i} \cdot w + \boldsymbol{pA_i} \qquad (i = 2, 3, \dots, n) \end{aligned} \tag{3}$$

In terms of the value of capital in sector 1, all prices become:

<u>Stage I:</u> Only r_1 increases from 0 to R; all other r_i's=0.

$$\frac{p_1}{\boldsymbol{pA_1}} = \frac{a_1 \cdot w}{\boldsymbol{pA_1}} + (1 + \boldsymbol{r}_1)$$

$$\frac{\boldsymbol{p_i}}{\boldsymbol{pA_1}} = \frac{\boldsymbol{a_i} \cdot w}{\boldsymbol{pA_1}} + \frac{\boldsymbol{pA_i}}{\boldsymbol{pA_1}} \qquad (i \neq 1) \tag{4}$$

Stage II: When $r_1 = R$, $r_2 > 0$, all other r_i's $(i > 2) = 0$.

and similarly for all other $i > 2$ when r_2 has reached its maximum value R.

$$\frac{p_1}{\boldsymbol{pA_1}} = \frac{a_1 \cdot w}{\boldsymbol{pA_1}} \; on \; (1 + R)$$

$$\frac{p_2}{\boldsymbol{pA_1}} = \frac{a_2 \cdot w}{\boldsymbol{pA_1}} + (1 + r_2) \frac{\boldsymbol{pA_2}}{\boldsymbol{pA_1}} \tag{5}$$

$$\frac{\boldsymbol{p_i}}{\boldsymbol{pA_1}} = \frac{\boldsymbol{a_i} \cdot w}{\boldsymbol{pA_1}} + \frac{\boldsymbol{pA_i}}{\boldsymbol{pA_1}} \qquad (i > 2)$$

It may be helpful, at this Stage, to consider an explicit analysis of the case in which i=3. Setting $\mathbf{pA}_1 = 1$, we could rewrite the two equations in (3) as:

Stage I:

$$p_1 = a_1 \cdot w + (1 + r)$$

$$\left[p_2 , p_3\right] = \left[a_2 , a_3\right] w + (a_1 \cdot w + (1+r)) \cdot \left[a_{12} , a_{13}\right] + \left[p_2 , p_3\right] \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \quad \textbf{(6)}$$

$$= \left(\left[a_2 , a_3\right] w + \left(a_1 \cdot w + (1+r)\right)\left[a_{12} , a_{13}\right]\right) \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1}$$

Since $\mathbf{pA}_1 = 1$, we have:

$$\left(a_1 \cdot w + (1+r)\right) a_{11} + \left(\left[a_2 , a_3\right] w + \left(a_1 \cdot w + (1+r)\right) \left[a_{12} , a_{13}\right]\right) \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1} \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix} = 1 \quad \textbf{(7)}$$

Rearranging and solving for w we get:

$$\frac{1 - (1+r) \cdot \left(a_{11} + \left[a_{12} , a_{13}\right] \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1} \cdot \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix} \right)}{a_1 a_{11} + \left(\left[a_2 , a_3\right] + a_1 \left[a_{12} , a_{13}\right]\right) \cdot \left[I - \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \right]^{-1} \cdot \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix}} = w \quad \textbf{(8)}$$

Results

It can be seen from equation (8) that the resulting wage-profit curve for this Stage I is a negatively sloped ***straight line***. We must interpret this result as saying that in terms of the value of capital in sector 1, the "capital intensities" in both the other sectors remain invariant to changes in the distribution of income. Nevertheless since, in terms of the wage rate, all prices increase, but r changes only in sector 1, the price of that sector's commodity, in terms of the numéraire increases faster than all others, so that all other prices must decrease or at least, must not increase.

From the linearity of the wage-profit curve in this Stage I we can write:[1]

$$\dot{w} = -\frac{\frac{1}{v}}{R} = -\frac{\frac{1}{a_1}}{R+1} \qquad \textbf{(9)}$$

From the derivative of p_1 with respect to r we get $\dot{p}_1 = a_1 \dot{w} + 1 > 0$ given that $-\dot{w} < \frac{1}{a_1}$ Substituting this condition in equation (9) we have $\frac{1}{a_1} > \frac{1}{a_1(R+1)}$ which is of course true for all values of $R \neq 0$.[2] In the case of p_2, from its derivative with respect to r it can be seen a priori that $\dot{p}_2 < 0$ and also linear.[3]

More generally, we may write, for $\mathbf{pA}_1 = 1$ in Stage I:

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1. See Steedman [1988] for the relationship between v^{-1} and a^{-1}, and thus R and R+1 in this equation.

2. Note that for negative values of R equation (9) is rewritten as $\dot{w} = +\frac{\frac{1}{a_1}}{1-R}$ to account for the positive slope. As it can easily be verified, equation (9) would still be satisfied but with the above expression in the right hand side of the inequality.

3. Note that from (3): $\dot{p}_2 = a_2 \cdot \dot{w}$ where the dot signifier than partial derivative with respect to r.

$$\left[p_1, p_2, \ldots, p_n, w \right] \begin{bmatrix} a_{11} & 1 & -a_{12} & \cdots & -a_{1n} \\ a_{12} & 0 & (1-a_{22}) & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -a_1 & -a_2 & \cdots & -a_n \end{bmatrix} = \left[1, (1+r), 0, \ldots, 0 \right] \tag{10}$$

Postmultiplying the right hand side vector by the inverse of the left hand side matrix, we notice that, since the rate of profit does not appear inside that matrix, the resulting prices and wage rate are a ***linear*** function of the rate of profit. Moreover, only the first two rows of that inverse are relevant for the determination of prices, ***independently of the number of commodities considered***,[4] while it can be seen that the signs of the elements of the second row will be positive for the first column and negative for all the others, thus making p_1 a linearly ***increasing*** function of the rate of profit r_1, while p_2, ..., p_n and w will all be linear but ***decreasing*** functions.

4. This is because of the addition of more zeros in the right hand side vector.

In terms of what happens in Stage II, we have:

Stage II:

$$p_1 = a_1 \cdot w + (1 + R)$$

$$p_2 = a_2 \cdot w + (1+r)\left[p_1 , p_2 , p_3 \right] \cdot \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} \tag{11}$$

$$p_3 = a_3 \cdot w + \left[p_1 , p_2 , p_3 \right] \cdot \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

Following a similar procedure as for Stage I and keeping the value of capital in sector 1 as our numéraire, we obtain, for w:

$$\frac{1 - (1+R) \cdot \left(a_{11} + \left[(1+r)a_{12} , a_{13} \right] \left[I - \begin{bmatrix} (1+r)a_{22} & a_{23} \\ (1+r)a_{32} & a_{33} \end{bmatrix} \right]^{-1} \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix} \right)}{a_1 \, a_{11} + \left(\left[a_2 , a_3 \right] + a_1 \left[(1+r)a_{12} , a_{13} \right] \right) \left[I - \begin{bmatrix} (1+r)a_{22} & a_{23} \\ (1+r)a_{32} & a_{33} \end{bmatrix} \right]^{-1} \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix}} = 1 \tag{12}$$

Upon inspection it can be verified that the negatively sloped wage profit curve obtained for Stage II of our analysis is ***not*** a straight line, in general, so that now, in terms of the value of capital in sector 1, the capital intensities of the other sectors will indeed vary as the rate of profit is increased.

A Generalization

To further isolate the movement in relative prices in this Stage, from the observed movements in relative values of capital, we should re-express the system of prices in terms of the value of capital of sector two, i.e. using as numéraire $\mathbf{pA}_2$=1. Following the same procedure as that suggested in equation (11) we may now write:

$$\left[p_1, p_2, ..., p_n, w\right] \begin{bmatrix} a_{12} & 1-\rho a_{11} & 0 & \ldots & -a_{1n} \\ a_{22} & -\rho a_{21} & 1 & \ldots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -a_1 & -a_2 & \ldots & -a_n \end{bmatrix} = \left[1, 0, (1+r), ..., 0\right. \qquad (13)$$

where ρ = (1+R), and from which we can see, that it will be the first and third row of the inverse matrix which will be relevant in the determination of relative prices. Furthermore, as was the case in Stage I (when we used $\mathbf{pA}_1$ as numéraire), when we use as numéraire the value of capital of the commodity for which the rate of profit changes, prices become ***linear***, and from the elements in the third row of the inverse matrix, only p_2 is an increasing function of the rate of profit, while all the others prices ($p_i \neq p_2$) and w are decreasing functions of the rate of profit. It should be clear from the above equation that these results apply for the general case of "**n**" commodities, as in Stage I.

This alternative procedure has enabled us to separate not only the effect of the change in the rate of profit in each sector individually, but also, it has eliminated the efects of the change in the relative values of capital, as the rate of profit is increased, sector by sector. Of course, we can always go back to the uniform numéraire if at every Stage we multiply the resulting price vectors by the relative $\frac{pA_i}{pA_j}$ where **j** is the sector whose rate of profit is changed. Note also that the prices used for the computation of these values of capital are the ones obtained by this process of increasing the rate of profit, sector by sector.

Moreover we know that the values of capital are affected by the particular behaviour of the overall vector of prices, weighted by the elements of the coefficient matrix. Thus, although we cannot, at this Stage make any specific statement about the movement of these relative values of capital, we can indeed say that we should expect these to change less than the respective relative prices.

Note that in the common analysis of changes in the system of relative prices, linear wage-profit curves meant that relative prices remained constant. In our case, what remains constant ***during Stage I of the analysis*** is the value of capital per worker, since we have taken the value of capital in the only sector for which the rate of profit changes to be equal to one, while in the other sectors r does not changes by construction. On the other hand relative prices do change, because although relative capital intensities remain the same, r changes only in sector 1, thus increasing its price faster than all others.

An Empirical Application

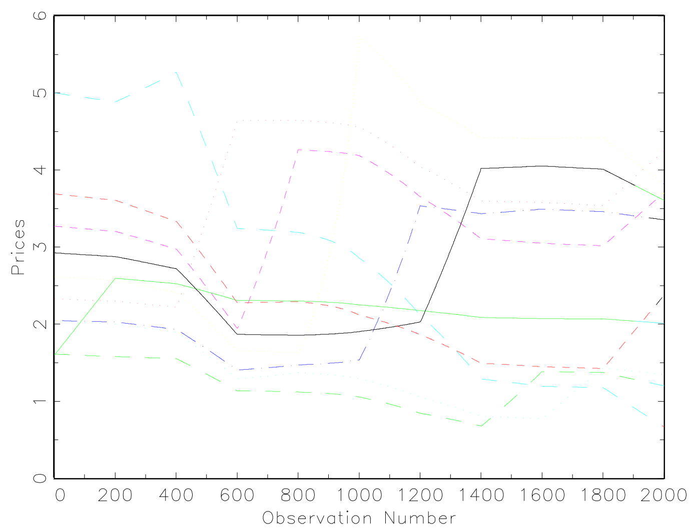
In figures1 and 2 we present the empirical results of this exercise for cases two. Since the point of the exercise is to see whether we can detect some simpler behaviour of the vector of prices as the rate of profit is increased in this step-by-step fashion, we will use as an example a 10 by 10 input-output matrix for the Puerto Rican economy for the year 1967 because it was the year for which the most non-monotonic price vectors were found under a uniform rate of profit scenario (del Valle, 1992, Chapter 5). For the sake of simplicity, we aggregated the 1967 transactions matrix (Puerto Rico Planning Board) to 10 sectors - following a general but arbitrary classification between agriculture, manufacturing, commerce, services and government activities- and computed the new coefficient matrix. Then we rearranged the rows and columns so as to put first those sectors which we thought had more interindustrial linkages with the other sectors. (As an indicator of these linkages we used the rank order of the column sums of Pasinetti's (1973) **H** matrix). The order of the sectors was selected in order to maximize the effects of the accumulated increases in the rates of profit during the first couple of "iterations". In this way we expected to observe the most significant changes in relative prices in the first few steps of increasing one r_j at a time.

We computed the new eigenvalue for the reduced 10 x 10 matrix and hence the new maximum rate of profit, which we would use to compute the range of the values of the rate of profit and the system of relative prices in terms of the standard wage. With the computed prices we analysed, as we said before, the individual movement of the various price vectors.

As we can see the behaviour of the relative prices has been greatly simplified in two major ways. First, prices have become more "linear" than in the common computation from actual empirical analyses, becoming ***perfectly linear*** in Stage I. Secondly, it can be seen that the changes in each price are mostly due to changes in its own rate of profit, and that once this has been accounted for, some prices tend to become "nearly-constant", the "non-linearity" being brought to the system by the continuous change in the relative value of capital as the rate of profit changes sector by sector. That is to say that they are not much affected by increases in the rate of profit in other sectors.

FIGURE 1[5]

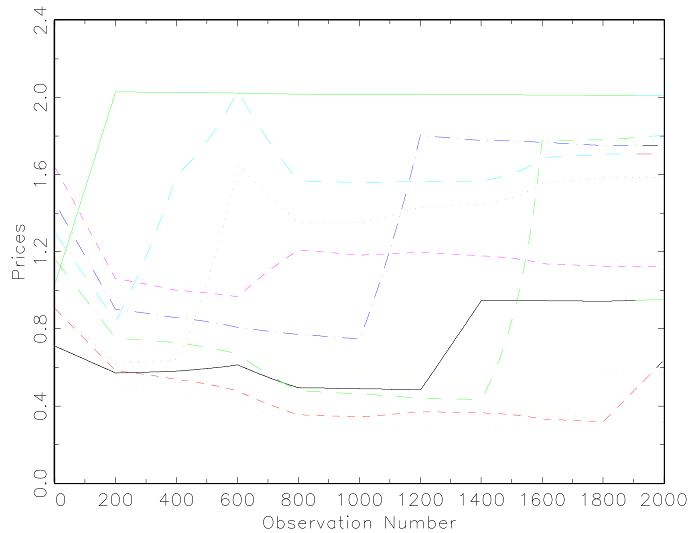
Relative Prices using the value of capital of sector 8 as nùmeraire



5. The increase in each sector's rate of profit was computed as R/200, so that in the horizontal axis we refer to "Observation Number" to mark the "Stage" of the analysis. In this way Stage I goes from observation 0 to 200, Stage II goes from observation 201 to 400, and so forth.

FIGURE 2

Relative Prices using the value of capital of sector 4 as nùmeraire



Concluding Remarks

Since the system of prices has been placed at the centre of the explanation of the behaviour of the capital-output and capital-labour ratios, in this paper we put forward an alternative analysis of the behaviour of the price system. Here we have tried to show that the behaviour of prices could be greatly simplified if we could "disect" or "separate" (i) the effect of the individual, industry by industry, changes n the rate of profit from the whole system of prices, and (ii) the effects of changes in the system of prices from the changes in the relative values of capital.

To do this we formalized a "Two-Stage" model in which, in the first stage, the rate of profit is increased only in industry "1"; all other industries' rate of profit being zero, and using as numéraire the value of capital in that industry. During "Stage II", it was the second industry's rate of profit which was allowed to increase, keeping constant that of industry "1", while all others industries' rate were still kept equal to zero. For this second stage we took, alternatively, the value of capital in the first industry, and later on, the value of its own capital. It was shown by this exercise that when we change the rate of profit in only one industry at a time and use that industry's value of capital as numéraire, the wage-profit curves become ***perfectly linear*** and only the price of that industry whose rate of profit was allowed to change increased. All other prices were also linear but non-increasing.

On the other hand, when we went on to "Stage II" of the analysis, keeping as numéraire the value of capital in the first industry, it was shown that the wage-profit curves are ***not*** linear, so that relative prices will also change, although not necessarily in a monotonic way, given the fact that now relative capital intensities will also change as the rate of profit is varied. Nevertheless we could say that even in this second case, since the rate of profit is still changing in only one industry, change in the relative values of capital reflect only the behaviour caused by the changes system of prices brought about by changes in that industry's rate of profit. Since we know from the literature that the behaviour of the value of capital is a result of the behaviour of the system of prices "weighted" by the relative importance of the elements of matrices **A** or **H**, we expect the change in the relative value of capital to be smaller than the change in the system of relative prices.

We made our analysis in a formal way, but by way of two examples we showed how in a first Stage the wage-profit curve was a straight line with prices changing in a linear way, with only the price of the sector for which the profit rate increased, increasing. At further Stages of the analysis, the behaviour of the prices was simpler if only because the price of the commodity for which the rate of profit changed increased faster than the others, while now other commodities prices' could also increase, because relative values of capital changed, given that the wage-profit curve was not a straight line.

When we changed the numéraire according to the sector for which the profit rate changed, (or according to the "Stage" of the analsysis), we observed that the linearity of the changes in ***all*** prices was retained, as well as the uniquenes of the increasing nature of the particular price whose rate of profit was changed. All our results were generalized to an "n" commodities model.

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